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ON SOME TESTS OF HOMOGENEITY OF VARIANCES

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Summary

In their paper [1], Ansari and Bradley discussed a two-sample rank test for dispersions and suggested the desirability of extending their results to the problem of several samples. In this paper, besides generalizing their results, we provide a few additional non-parametric tests, which include, among others, the multi-sample analogues of the two-sample normal scores test of dispersion and the tests considered by Mood [6], Klotz [4], and Siegel and Tukey [10]. The asymptotic distributions of the proposed test statistics are derived by an application of the author's theorem [7]. The asymptotic efficiencies of these tests relative to one another and the $\mathcal{F}$ test ([9] pp. 83-87) are computed in the standard fashion along the lines of the author's paper [7].

1. Introduction

Let $X_{ij}(j=1,\ldots,m_i; i=1,\ldots,c)$ be independent samples from populations with continuous cumulative distribution function $F_i(x) = F(\theta_i(x-\nu)$, where θ_i and ν are real numbers. θ_i is the scale parameter for F_i and ν, the common median for all F_i, which without loss of generality can be taken to be zero. We are interested in testing the hypothesis $H_o : \theta_1 = \ldots = \theta_c$ against $H_1 : \theta_i \neq \theta_j$ for some pair (i,j).

Let $Z_{N,i}^{(j)} = 1$, if the ith smallest observation from the combined sample of size $N = \sum m_i$ is from the jth sample and otherwise let $Z_{N,i}^{(j)} = 0$. Then we propose to consider the following test statistics.

A. $$\mathcal{L}(B) = 48 \sum_{j=1}^{c} m_j(B_{N,j} - \mathcal{B}_{N,j})^2$$

where

(1.1) $$m_j B_{N,j} = \sum_{i=1}^{N} \left[\frac{1}{2} + \frac{1}{2N} - \left|\frac{1}{2} + \frac{1}{2N} - \frac{i}{N}\right| \right] Z_{N,i}^{(j)}$$

B. $$\mathcal{L}(M) = 180 \sum_{j=1}^{c} m_j(M_{N,j} - \mathcal{C}_{N,j})^2$$

where

(1.2) $$m_j M_{N,j} = \sum_{i=1}^{N} \left(\frac{i}{N} - \frac{N+1}{2N} \right)^2 Z_{N,i}^{(j)}$$

C. $$\mathcal{L}(\Psi) = \sum_{j=1}^{c} m_j \left[(\Psi_{N,j} - C_{N,j})/A_N\right]^2$$

where

$$(1.3) \qquad m_j \Psi_{N,j} = \sum_{i=1}^{N} E_\Psi \left[V^{(i)}\right]^2 Z_{N,i}^{(j)}$$

and where $V^{(1)} < \ldots < V^{(N)}$ is an ordered sample of size N from a distribution Ψ and E denotes the expectation. ($\mathcal{B}_{N,j}, \mathcal{U}_{N,j}, C_{N,j}$ and A_N are normalizing constants to be defined below).

We may note that with $c = 2$, the statistics $\mathcal{L}(B)$, $\mathcal{L}(M)$ and $\mathcal{L}(\Phi)$ where Φ is the standard normal distribution function, reduce respectively to the two-sided Ansari-Bradley-Freund W statistic [1], Mood's M statistic [6] and the normal scores statistic [3], for testing the equality of dispersions of the two populations.

Reference to prior work on the two-sample non-parametric tests for dispersions may also be found in Lehmann [5], Sukhatme ([11], [12]), Barton and David [2], Siegel and Tukey [10], and Klotz [4], among others. Ansari and Bradley [1] have shown that their W test is equivalent to one independently proposed by Barton and David [2]. Klotz [4] has shown that the W test is equivalent to one proposed by Siegel and Tukey [10]. Thus the $\mathcal{L}(B)$ test, defined above, may also be regarded

as a generalization of the two-sided Ansari-Bradley-Barton-David-Siegel-Tukey test of dispersion. Lehmann's test is not a distribution-free test and so the relative efficiencies of this test are not known. Sukhatme's second test [12] is an improvement over the Ansari-Bradley W test and its generalization to several samples merit investigation; but since this does not appear to be possible with the methods of the present paper, we do not intend to discuss this test here. We shall confine our attention to the statistics proposed in A, B, and C above. (The author is not aware of any multi-sample rank test for dispersions in the literature).

In the passing, we may remark that the statistics proposed above may be put in the general framework of the statistics $\mathcal{L}$ defined as

$$\mathcal{L} = \sum_{j=1}^{c} m_j \left[(T_{N,j} - \mu_{N,j})/A_N\right]^2 \tag{1.4}$$

where $\mu_{N,j}$ and A_N are normalizing constants, and

$$m_j T_{N,j} = \sum_{i=1}^{N} E_{N,i} \, Z_{N,i}^{(j)} \quad . \tag{1.5}$$

With $E_{N,i} = \frac{1}{2} + \frac{1}{2N} - \left|\frac{1}{2} + \frac{1}{2N} - \frac{i}{N}\right|$, the statistic $\mathcal{L}$

reduces to the $\mathcal{L}(B)$ statistic. With $E_{N,i} = \left(\frac{i}{N} - \frac{N+1}{2N}\right)^2$ we obtain the $\mathcal{L}(M)$ statistic and with $E_{N,i} = E_{N,i} = E_{\Psi}\left[V^{(i)}\right]^2$ we obtain the $\mathcal{L}(\Psi)$ statistic. The tests based on the statistics $\mathcal{L}(B)$, $\mathcal{L}(M)$ and $\mathcal{L}(\Psi)$ will be referred as the $\mathcal{L}$ tests.

The statistics $\mathcal{L}$, defined by (1.4) with weights $E_{N,i}$ different than the ones proposed above, were suggested by the author [7] for testing the equality of location parameters of the c probability distributions. Here we shall find the limiting distributions of the statistics $\mathcal{L}$ (defined by A, B, and C) when the distributions $F_i(x)$ differ only by scale parameters. The formulation of the problem as given in this paper is the same as given in the earlier paper. However, the earlier paper treats the problem somewhat more generally, and the reader is referred to it for additional background and motivation.

2. The limiting distributions of $\mathcal{L}$ under local alternatives.

To obtain the large sample distribution of the statistics $\mathcal{L}$, we begin by determining the asymptotic distributions of the statistics $T_{N,j}$ defined by (1.5). This is given by the following theorem, where the sample sizes m_i are assumed to tend to infinity in such a way that $m_i = s_i \cdot n$, $n \to \infty$; $i=1,\ldots,c$.

Theorem 2.1. <u>For</u> $n = 1,2,\ldots$ <u>let</u> $X_{ij}(j=1,\ldots,m_i(n);$ $i=1,\ldots,c)$ <u>be independently distributed according to</u> $F(\theta_i^{(n)}x)$. <u>Suppose that the sequence of parameter points</u>

$\vartheta^{(n)} = (\vartheta_1^{(n)},\ldots,\vartheta_c^{(n)})$ satisfy

(2.1) $$\vartheta_i^{(n)} = 1 + \nu_i/\sqrt{n} \quad .$$

Let the assumptions of theorem 6.2 of [7] be satisfied. Then the variables $(W_1,\ldots,W_c)$ given by

(2.2) $$W_j = m_j^{1/2}\left(T_{N,j} - \mu_{N,j}(\vartheta^{(n)})\right)$$

have a joint asymptotic normal distribution with zero means and covariance matrix whose (j,j')th term is

(2.3) $$\left(\delta_{jj'} - \frac{\sqrt{s_j s_{j'}}}{\sum s_i}\right)A^2$$

where the $\delta_{jj'}$ are the kronecker deltas, and, where

(2.4) $$A^2 = \int_0^1 (\Psi^{-1}(x))^4 dx - \left(\int_0^1 (\Psi^{-1}(x))^2 dx\right)^2 \text{ for the } \mathcal{L}(\Psi) \text{ test.}$$

$$= \frac{1}{48} \text{ for the } \mathcal{L}(B) \text{ test.}$$

$$= \frac{1}{180} \text{ for the } \mathcal{L}(M) \text{ test.}$$

Note that, under the assumptions of Theorem 2.1,

we have (cf. [7])

$$(2.5)\quad \mu_{N,j}(\theta^{(n)}) = \int_{-\infty}^{+\infty} \left[\Psi^{-1}(H(x))\right]^2 dF_j(x) \text{ for the } \mathcal{L}(\Psi) \text{ test.}$$

$$= \int_{-\infty}^{0} H(x)dF_j(x) + \int_{0}^{\infty} (1-H(x))dF_j(x)$$

for the $\mathcal{L}(B)$ test.

$$= \int_{-\infty}^{+\infty} \left(H(x) - \tfrac{1}{2}\right)^2 dF_j(x) \text{ for the } \mathcal{L}(M) \text{ test.}$$

where

$$(2.6)\qquad F_j(x) = F\left(x(1 + \nu_j/\sqrt{n}\right)$$

and

$$(2.7)\qquad H(x) = \sum_{i=1}^{c} s_i F_i(x) \Big/ \sum_{\alpha=1}^{c} s_\alpha \quad .$$

The proof of the theorem follows as did theorem 7.1 in [7] from theorem 6.2 of that paper. Now making the analysis of variance transformation

$$s_o = \sum_{i'=1}^{c} e_{i'}^{1/2} W_{i'}/A \quad \text{where} \quad e_{i'} = s_{i'}\Big/\sum_{i=1}^{c} s_i$$

$$s_i = \sum_{i'=1}^{c} a_{ii'} W_{i'}/A \quad ; \quad i=1,\ldots,c-1$$

where the a's are chosen to make the transformation orthogonal and proceeding as in [7], we arrive at the following:

Theorem 2.2. <u>Suppose that the assumptions of Theorem</u> 2.1 <u>are satisfied</u>.

(a) <u>If</u>

$$(2.8) \qquad \lim_{n\to\infty} \sqrt{m_j}\left(C_{N,j}(\theta^{(n)}) - C_{N,j}(1)\right)/A$$

<u>exists and is finite, then, for</u> $n \to \infty$, <u>the statistic</u> $\mathcal{L}(\Psi)$ <u>has a limiting non-central chi-square distribution with</u> $c-1$ <u>degrees of freedom and non-centrality parameter</u>

$$(2.9) \qquad \lambda_\Psi = \sum_{j=1}^{c}\left[\lim_{n\to\infty} \sqrt{m_j}\left(C_{N,j}(\theta^{(n)}) - C_{N,j}(1)\right)\right]^2/A^2 \quad .$$

(b) <u>If</u>

$$(2.10) \qquad \lim_{n\to\infty} \sqrt{m_j}\left(\mathcal{B}_{N,j}(\theta^{(n)}) - \mathcal{B}_{N,j}(1)\right)$$

<u>exists and is finite, then, for</u> $n \to \infty$, <u>the statistic</u> $\mathcal{L}(B)$ <u>has a limiting non-centrality chi-square distribution with</u>

c-1 degrees of freedom and non-centrality parameter

$$(2.11) \qquad \lambda_B = 48 \sum_{j=1}^{c} \left[\lim_{n \to \infty} \sqrt{m_j} \left(\mathcal{B}_{N,j}(\theta^{(n)}) - \mathcal{B}_{N,j}(1) \right) \right]^2 .$$

(c) If

$$(2.12) \qquad \lim_{n \to \infty} \sqrt{m_j} \left(\mathcal{A}_{N,j}(\theta^{(n)}) - \mathcal{A}_{N,j}(1) \right)$$

exists and is finite, then, for $n \to \infty$, the statistic $\mathcal{L}(M)$ has a limiting non-central chi-square distribution with c-1 degrees of freedom and non-centrality parameter

$$(2.13) \qquad \lambda_M = 180 \sum_{j=1}^{c} \left[\lim_{n \to \infty} \sqrt{m_j} \left(\mathcal{A}_{N,j}(\theta^{(n)}) - \mathcal{A}_{N,j}(1) \right) \right]^2 .$$

Corollary. If, in addition to the assumptions of Theorem 2.2, the regularity conditions of lemma 7.2 of [9] are satisfied, then,

$$(2.14) \qquad \lambda_\Psi = \sum_{j=1}^{c} s_j(\nu_j - \bar{\nu})^2 \left(\int_{-\infty}^{+\infty} x \frac{d}{dx} \left\{ \Psi^{-1}[F(x)] \right\}^2 dF(x) \right)^2 \Big/ A^2$$

$$(2.15) \qquad \lambda_B = 48 \sum_{j=1}^{c} s_j(\nu_j - \bar{\nu})^2 \left(\int_{-\infty}^{0} x \, f^2(x)dx - \int_{0}^{\infty} x \, f^2(x)dx \right)^2$$

$$(2.16) \qquad \lambda_M = 12 \sum_{j=1}^{c} s_j(\nu_j - \overline{\nu})^2 \left(\int_{-\infty}^{+\infty} x\ f(x)[2F(x)-1]dF(x) \right)^2$$

<u>where</u> f <u>is the density of</u> F <u>and</u>

$$(2.17) \qquad \overline{\nu} = \sum s_i \nu_i / \sum s_i \quad .$$

Note: To get λ_B, we need a trivial modification of the part (i) of lemma 7.2 of [9].

Proof. To get (2.14) from (2.9), we note that

$$= \sqrt{n}\left(C_{N,j}(\theta^{(n)}) - C_{N,j}(1)\right)$$

$$= \sqrt{n}\left(\int J\left[\sum_{i=1}^{c} \lambda_i F\left(x(1+\nu_i/\sqrt{n})\right)\right] dF\left(x(1+\nu_j/\sqrt{n})\right) - \int J\ [F(x)]dF(x)\right)$$

$$= \int A_n(x)B_n(x)dF(x)$$

where

$$A_n(x) = \frac{J\left[\sum_{i=1}^{c} \lambda_i F\left(x\left(1+\nu_i/\sqrt{n}\right)\left(1+\nu_j/\sqrt{n}\right)^{-1}\right)\right] - J[F(x)]}{\sum_{i=1}^{c} \lambda_i F\left(x\left(1+\nu_i/\sqrt{n}\right)\left(1+\nu_j/\sqrt{n}\right)^{-1}\right) - F(x)}$$

$$B_n(x) = \sqrt{n}\sum_{i=1}^{c} \lambda_i\left[F\left(x\left(1+\nu_i/\sqrt{n}\right)\left(1+\nu_j/\sqrt{n}\right)^{-1}\right) - F(x)\right] \quad .$$

Under the assumed regularity conditions

$$\lim_{n\to\infty} A_n(x) = \frac{d}{du} J(u)\Big|_{u = F(x)}$$

$$\lim_{n\to\infty} B_n(x) = (\overline{\nu} - \nu_j)\, x\, f(x)$$

where $\overline{\nu}$ is given by (2.17). Since differentiation under the integral is permitted, the proof follows.

The proofs of (2.15) and (2.16) are exactly analogous.

In particular when $\Psi = \Phi$, where Φ is the standard normal distribution function,

$$\text{(2.18)} \qquad \lambda_{\Phi} = 2 \sum_{j=1}^{c} s_j(\nu_j - \overline{\nu})^2 \left(\int \frac{x\, \Phi^{-1}[F(x)] f(x) dF(x)}{\phi\left[\Phi^{-1}[F(x)]\right]} \right)^2$$

where ϕ is the density of Φ.

3. Parametric Case

In the parametric theory, a commonly used test is Bartlett's H test for the homogeneity of variances. This test is shown by Box to be very sensitive to departures from normality assumptions. We give below a very brief description of an approximate test based on the analysis of variance of the logarithms of the sample variances. For details, the reader is referred to Scheffé ([9], pp. 83-87), from where the following

portion is taken with slight modification.

Divide the ith sample $X_{i1},\ldots,X_{in_i}$ into J_i sub-samples. Let m_{ij} denote the size and s_{ij}^2 the sample variance of the jth sub-sample of the ith sample. Denote $Y_{ij} = \log s_{ij}^2$ and assume that γ_2, the kurtosis measure has the same value for all populations. Then from Scheffé (cited above), the test for the hypothesis of the equality of variances is based on the statistic

$$(3.1) \qquad \mathcal{F} = \frac{\frac{1}{c-1}\sum_i \nu_i^*(\hat{\eta}_i - \hat{\eta})^2}{\frac{1}{\nu_e}\sum_i\sum_j \nu_{ij}(Y_{ij} - \hat{\eta}_i)^2}$$

where $\nu_{ij} = m_{ij} - 1$, $\nu_i^* = \sum_{j=1}^{J_i} \nu_{ij}$, $\hat{\eta}_i = \sum_j \nu_{ij}Y_{ij}/\nu_i^*$,

$\hat{\eta} = \sum_i \nu_i^*\hat{\eta}_i/\nu^*$, $\nu^* = \sum_i \nu_i^*$ and $\nu_e = \sum_i (J_i - 1)$. The

denominator of $\mathcal{F}$ converges to $2+\gamma_2$ in probability as $n \to \infty$. (Note that we are assuming $m_{ij} \to \infty$ as $n \to \infty$). Hence an asymptotically equivalent statistic is

$$(3.2) \qquad (c-1)\mathcal{F} = \sum_i \nu_i^*(\hat{\eta}_i - \hat{\eta})^2/(2+\gamma_2)$$

which has a limiting non-central chi-square distribution with a non-centrality parameter (after omitting computations)

$$(3.3) \qquad \lambda_{\mathcal{T}} = 4 \sum_{i=1}^{c} s_i(\nu_i - \bar{\nu})^2 \Big/ (2+\gamma_2) \quad .$$

4. Asymptotic Relative Efficiency.

It is well-known ([7]) that in the situations we are considering, the asymptotic efficiency of one statistic relative to another is equal to the ratio of their non-centrality parameters. Hence, we have the efficiencies of the normal scores $\mathcal{L}(\Phi)$ statistic relative to $\mathcal{L}(B)$, $\mathcal{L}(M)$ and $\mathcal{L}(\mathcal{T})$ statistics as follows:

$$(4.1) \qquad e_{\mathcal{L}(\Phi),\mathcal{L}(B)} = \lambda_{\Phi}/\lambda_B$$

$$(4.2) \qquad e_{\mathcal{L}(\Phi),\mathcal{L}(M)} = \lambda_{\Phi}/\lambda_M$$

$$(4.3) \qquad e_{\mathcal{L}(\Phi),\mathcal{T}} = \lambda_{\Phi}/\lambda_{\mathcal{T}} \quad .$$

It may be remarked that the above results agree with the results obtained by Ansari and Bradley [1], and Klotz [4] for the two-sample case, and hence the results of this paper as well as of these authors apply directly to the c-sample problem. We would, in particular, draw the attention of the reader to Section 3 of [4] and Section 7 of [1] where

the efficiency comparisons for different densities of the $\mathcal{L}(\Phi)$, $\mathcal{L}(M)$, $\mathcal{L}(B)$ and $\mathcal{J}$ tests are made for the two-sample problem and which are shown here to be valid also for the c-sample problem.

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13. ABSTRACT

For testing the equality of c continuous probability distributions on the basis of c independent random samples, various non-parametric tests for dispersions are offered. These tests, include among others, the multi-sample analogues of the two-sample normal-scores test of dispersion and the tests considered by Ansari and Bradley (Ann. Math. Statist. 1960), Klotz (Ann. Math. Statist. 1962), Mood (Ann. Math. Statist. 1954), and Siegel and Tukey (J. Amer. Stat. Assn. (1960)). Under suitable regularity conditions, it is shown that these tests have limiting non-central chi-square distributions with c-1 d.f. The asymptotic relative efficiencies (in the Pitman sense) of these tests relative to one another and the $\mathcal{F}$ test (Scheffe: The Analysis of Variance, pp. 83-87) are obtained and are shown to be the same as for the two-sample problem [cf. Ansari-Bradley and Klotz cited above].

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